Harmonic Distance Functions

Several theorists (Barlow, Tenney, Euler, and others) have attempted to devise functions which measure, in some explainable way, relative consonance, or perhaps more accurately, the complexity, of intervals. These functions usually have two main criteria for "harmonicity" (or perhaps, consonance, though this is can be a stylistic/cultural term as well):

1) smallness of primes
2) smallness of prime powers

In other words, such a function typically finds $3/2$ more consonant than $5/4$, $5/4$ than $7/6$, but has some trouble distinguishing between, for example $10/8$ and $9/8$ (both major seconds, one with a higher prime power of $3^2$, the other with a higher prime (5) in the numerator). In general, powers of 2 don't count for much.
A simple vector representation of a ratio

\[
\begin{align*}
5/4 &= \{201\} \\
3/2 &= \{11\} \\
15/8 &= \{311\} \\
40/27 &= \{331\} \text{ (the "wolf fifth")}
\end{align*}
\]

— where the nth place in the vector is the power of the nth prime present in the ratio (positive for numerator, negative for denominator). In practice, 2 (the oddest prime) can be omitted.

This representation is often used in computation, and computations of harmonic distance. Simple vector addition corresponds to ratio multiplication. Also, easy to see "harmonic complexity" by length of vector, magnitude of numbers.